

RESPONSE OF UNDAMPED SDOF SYSTEMS SUBJECTED TO TRIANGULAR PULSE

¹Rohit Gupta*, ²Rakesh Kumar Verma

¹Lecturer, Dept. of Applied Sciences (Physics), Yogananda College of Engg. and Tech., Jammu, J&K, India

²Assistant Professor, Dept. of Applied Sciences, Yogananda College of Engg. and Tech., Jammu, J&K, India

*Corresponding Author: guptarohit565@gmail.com

Received 26 June 2023 Received in Revised for 31 June 2023 Accepted 02 July 2023

ABSTRACT

The single degree of freedom (SDOF) systems have been studied usually via ordinary mathematical tools like calculus, for inquiring about the effects of various pulses like rectangular pulse, step force, triangular pulse, etc., on their responses. In this paper, the SDOF systems such as an undamped mechanical oscillator as well as an undamped electrical oscillator exposed to a triangular pulse have been contemplated and their responses have been determined by the integral Rohit transform (RT). The graphs of responses of SDOF systems versus time have been plotted. It is inferred from the graphs that even though the amplitude of triangular pulse is decreasing linearly, the response of SDOF systems is periodically increasing and decreasing. This paper put forward a new technique for determining the response of SDOF systems (undamped oscillators) exposed to a triangular pulse and reveals that RT is an effective tool for analyzing the SDOF systems.

Keywords: Rohit Transform (RT), undamped oscillator, triangular pulse, SDOF systems.

I. INTRODUCTION

The load F_0 is instantly applied to the structure and decreased linearly over time duration t_1 as shown in the figure below.

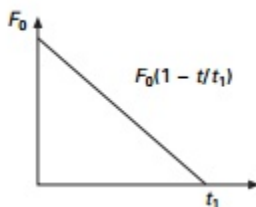


Figure 1: Triangular Pulse

The triangular pulse is written as:

$$F(t) = F_0 \left(1 - \frac{t}{t_1}\right) \text{ for } t < t_1$$

$$= 0 \text{ for } t \geq t_1.$$

A triangular pulse force is usually hired to spur a blast [1, 2]. It is becoming progressively important to

analyze and form structures to be safer against blast loads. Generally, it has been accepted that the analysis of single degree of freedom systems is best suited for such problems [3]. Therefore, in this paper single degree of freedom systems have been contemplated for inquiring about the effects of triangular pulse on the response of an undamped mechanical oscillator as well as an electrical oscillator exposed to a triangular pulse. The integral RT has been proposed by the author Rohit Gupta in recent years [4] to facilitate the process of solving ordinary and partial differential equations in the time domain. It has been applied to solve many initial value problems in science and engineering [5-7]. This paper put forward the RT for determining the response of an undamped mechanical oscillator as well as an electrical oscillator exposed to a triangular pulse. A Rohit transform is defined for a function of exponential order as follows: Considering functions in the set B defined as:

$$B = \{g(t): \exists R, q_1, q_2 > 0, |g(t)| < Re^{q_1 t}, \text{ if } t \in (-1)^i X [0, \infty)\}.$$

For a given function in set B, the constant R must be a finite number, q_1 and q_2 , may be finite or infinite. The Rohit transform of a function $g(t)$ [6] is defined by the integral equations as

$$R\{g(t)\} = q^3 \int_0^\infty e^{-qt} g(t) dt, t \geq 0, q_1 \leq q \leq q_2.$$

The variable q in this transform is used to factor the variable t in the argument of the function g .

The key motivation for applying RT for the dictation of the response of SDOF systems (an undamped mechanical oscillator as well as an undamped electrical oscillator) exposed to a triangular pulse is that the process of solving a governing ordinary differential equation for such problems is simplified to an algebraic problem. This method of converting the problems of calculus to algebraic problems is known as operational calculus. The RT method has two main advantages over the calculus method:

- i. Problems involving differential equations are worked out more directly i.e. initial (boundary) value problems are worked out without first ascertaining a general solution.
- ii. A non-homogenous differential equation is worked out without first working out the corresponding homogeneous differential equation.

The RT when applied to a function, changes that function into a new function by using a process that involves integration.

A unit step function [8] is written as $U(t - a) = 0$ for $t < a$ and 1 for $t \geq a$.

The Rohit transform of the unit step function is given by

$$R\{U(t - a)\} = q^3 \int_0^\infty e^{-qt} U(t - a) dt$$

$$R\{U(t - a)\} = q^3 \int_a^\infty e^{-qt} dt$$

$$R\{U(t - a)\} = q^2 e^{-qa}$$

Shifting property of Rohit transform:

If $R\{g(t)\} = G(q)$, then $R\{g(t - a)U(t - a)\} = e^{-qa}G(q)$.

Proof:

$$R\{g(t - a)U(t - a)\} = q^3 \int_0^\infty e^{-qt} g(t - a)U(t - a) dt$$

$$R\{g(t - a)U(t - a)\} = q^3 \int_a^\infty e^{-qt} g(t - a) dt$$

$$R\{g(t - a)U(t - a)\} = q^3 \int_0^\infty e^{-q(v+a)} g(v) dv,$$

where $v = t - a$

$$R\{g(t - a)U(t - a)\} = e^{-q(a)} q^3 \int_0^\infty e^{-q(v)} g(v) dv$$

$$R\{g(t - a)U(t - a)\} = e^{-q(a)} q^3 \int_0^\infty e^{-q(t)} g(t) dt$$

$$R\{g(t - a)U(t - a)\} = e^{-q(a)} G(q)$$

The RT of some basic functions is given by

$$\diamond R\{t^n\} = \frac{n!}{q^{n-2}},$$

where n is $0, 1, 2, \dots$

$$\diamond R\{\sin bt\} = \frac{b q^3}{q^2 + b^2},$$

$$\diamond R\{\cos bt\} = \frac{q^4}{q^2 + b^2},$$

The RT of some derivatives [4] of a function $g(t)$ is

$$R\{g'(t)\} = qR\{g(t)\} - q^3g(0),$$

$$R\{g''(t)\} = q^2R\{g(t)\} - q^4g(0) - q^3g'(0),$$

and so on.

Highlights

- ✓ Focusing on any research effort concerning literature review is the very paramount task because it builds up ideas that can evolve quickly. This paper focuses on the administration of Rohit transform and upto the minute integral transform technique.
- ✓ An RT technique is proposed for handling out the response of SDOF systems (an undamped mechanical oscillator as well as an undamped electrical oscillator) exposed to a triangular pulse. The differential equation representing the SDOF system is worked out more directly without first ascertaining a general solution.
- ✓ A non-homogenous differential equation representing the SDOF system is worked out without first working out the corresponding homogeneous differential equation.
- ✓ Graphs of responses with respect to time are plotted.
- ✓ Highly precise and accurate results are obtained.
- ✓ This paper shows beyond doubt that the integral transform RT is a potent mathematical tool for analyzing the SDOF systems.

II. MATERIAL AND METHOD

A. UNDAMPED MECHANICAL OSCILLATOR

The differential equation of the undamped mechanical oscillator [8] exposed to a triangular pulse force is given by

$$m\ddot{y}(t) + ky(t) = F_o \left(1 - \frac{t}{t_1}\right)$$

Or

$$\ddot{y}(t) + \omega_0^2 y(t) = \frac{F_o}{m} \left(1 - \frac{t}{t_1}\right) \dots (1)$$

where $\omega_0 = \sqrt{\frac{k}{m}}$, $F_o \left(1 - \frac{t}{t_1}\right)$ is a triangular pulse force, $y(0) = 0$ and $\dot{y}(0) = 0$.

The RT of (1) provides

$$q^2 \bar{y}(q) - q^4 y(0) - q^3 \dot{y}(0) + \omega_0^2 \bar{y}(q) = \frac{F_o}{m} q^3 \int_0^\infty e^{-qt} \left(1 - \frac{t}{t_1}\right) dt$$

$$\Rightarrow q^2 \bar{y}(q) - q^4 y(0) - q^3 \dot{y}(0) + \omega_0^2 \bar{y}(q) = \frac{F_o}{m} \left\{ q^3 \int_0^{t_1} e^{-qt} \left(1 - \frac{t}{t_1}\right) dt + q^3 \int_{t_1}^\infty e^{-qt} (0) dt \right\}$$

Here $\bar{y}(q)$ denotes the RT of $y(t)$.

Put $y(0) = 0$ and $\dot{y}(0) = 0$ and simplifying (2), we get

$$q^2 \bar{y}(q) + \omega_0^2 \bar{y}(q) = \frac{F_o}{m} \left\{ q^3 \int_0^{t_1} e^{-qt} (1) dt - q^3 \int_0^{t_1} e^{-qt} \left(\frac{t}{t_1}\right) dt \right\}$$

$$\Rightarrow q^2 \bar{y}(q) + \omega_0^2 \bar{y}(q) = \frac{F_o}{m} \left\{ -q^2 [e^{-qt_1} - 1] + q^2 [e^{-qt_1}] - \frac{q^2}{t_1} \int_0^{t_1} e^{-qt} dt \right\}$$

$$\Rightarrow q^2 \bar{y}(q) + \omega_0^2 \bar{y}(q) = \frac{F_o}{m} \left\{ -q [e^{-qt_1} - 1] + q^2 [e^{-qt_1}] + \frac{q}{t_1} [e^{-qt_1} - 1] \right\}$$

$$\Rightarrow q^2 \bar{y}(q) + \omega_0^2 \bar{y}(q) = \frac{F_o}{m} \left\{ q^2 + \frac{q}{t_1} e^{-qt_1} - \frac{q}{t_1} \right\}$$

$$\Rightarrow \bar{y}(q) = \frac{F_o}{m} \left\{ \frac{q^2}{(q^2 + \omega_0^2)} + \frac{q}{t_1 (q^2 + \omega_0^2)} e^{-qt_1} - \frac{q}{t_1 (q^2 + \omega_0^2)} \right\}$$

$$\Rightarrow \bar{y}(q) = \frac{F_o}{m} \left\{ \frac{q^4}{q^2 (q^2 + \omega_0^2)} - \frac{q^3}{t_1 q^2 (q^2 + \omega_0^2)} + \frac{q^3}{t_1 q^2 (q^2 + \omega_0^2)} e^{-qt_1} \right\}$$

$$\Rightarrow \bar{y}(q) = \frac{F_o}{m} \left\{ \frac{q^2}{(\omega_0^2)} - \frac{q^4}{(\omega_0^2)(q^2 + \omega_0^2)} - \frac{q}{t_1 (\omega_0^2)} + \frac{q^3}{t_1 (\omega_0^2)(q^2 + \omega_0^2)} + \frac{q}{t_1 (\omega_0^2)} e^{-qt_1} - \frac{q^3}{t_1 (\omega_0^2)(q^2 + \omega_0^2)} e^{-qt_1} \right\}$$

Taking inverse RT, we have

$$y(t) = \frac{F_o}{m} \left\{ \frac{1}{(\omega_0^2)} - \frac{\cos \omega_0 t}{(\omega_0^2)} - \frac{t}{t_1 (\omega_0^2)} + \frac{\sin \omega_0 t}{t_1 \omega_0 (\omega_0^2)} + \frac{(t - t_1)}{t_1 (\omega_0^2)} U(t - t_1) - \frac{\sin \omega_0 (t - t_1)}{t_1 \omega_0 (\omega_0^2)} U(t - t_1) \right\}$$

$$\Rightarrow y(t) = \frac{F_o}{m (\omega_0^2)} \left\{ 1 - \cos \omega_0 t - \frac{t}{t_1} + \frac{\sin \omega_0 t}{t_1 \omega_0} + \frac{(t - t_1)}{t_1} U(t - t_1) - \frac{\sin \omega_0 (t - t_1)}{t_1 \omega_0} U(t - t_1) \right\}$$

$$\Rightarrow y(t) = \frac{F_o}{k} \left\{ \frac{\sin \omega_0 t}{t_1 \omega_0} - \cos \omega_0 t + 1 - \frac{t}{t_1} + \frac{(t - t_1)}{t_1} U(t - t_1) - \frac{\sin \omega_0 (t - t_1)}{t_1 \omega_0} U(t - t_1) \right\}$$

$$\Rightarrow y(t) = \frac{F_o}{k} \left\{ \frac{\sin \omega_0 t}{t_1 \omega_0} - \cos \omega_0 t + \left(1 - \frac{t}{t_1}\right) - \left(1 - \frac{t}{t_1}\right) U(t - t_1) - \frac{\sin \omega_0 (t - t_1)}{t_1 \omega_0} U(t - t_1) \right\} \dots (3)$$

Taking $\frac{F_o}{k} = 1000$ m, $\omega_0 = \frac{314}{sec}$ rad, $t_1 = 1$ sec, the graph of equation (3) is shown in figure below.

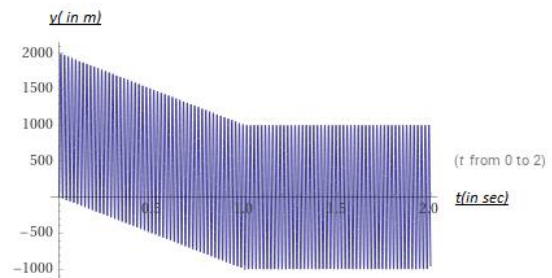


Figure 2: Response of an undamped mechanical oscillator exposed to a triangular pulse force

For $t < t_1$,

$$y(t) = \frac{F_0}{k} \left\{ \frac{\sin \omega_0 t}{t_1 \omega_0} - \cos \omega_0 t + \left(1 - \frac{t}{t_1}\right) \dots \dots (4a) \right.$$

Taking $\frac{F_0}{k} = 0.1m$, $t_1 = 0.1s$ and $\omega_0 = 314 \frac{rad}{s}$. The graph of equation (4b) is shown in figure 3.

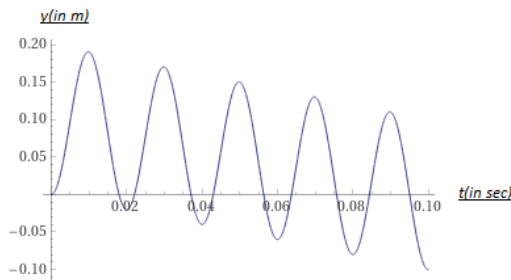


Figure 3: Response of an undamped mechanical oscillator during the application of triangular pulse force

For $t > t_1$,

$$y(t) = \frac{F_0}{k} \left\{ \frac{\sin \omega_0 t}{t_1 \omega_0} - \cos \omega_0 t - \frac{\sin \omega_0 (t-t_1)}{t_1 \omega_0} \right\} \dots \dots (4b)$$

Taking $\frac{F_0}{k} = 0.1m$, $t_1 = 0.1s$ and $\omega_0 = 314 \frac{rad}{s}$. The graph of equation (4b) is shown in Figure 4.

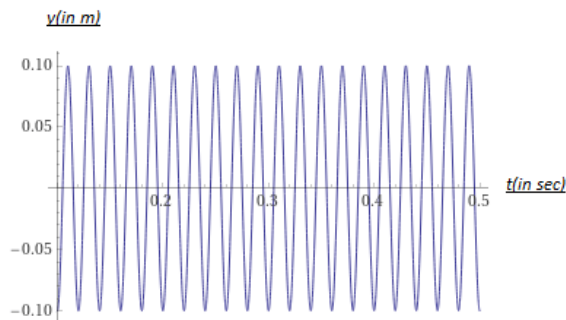


Figure 4: Response of an undamped mechanical oscillator after the termination of triangular Pulse force.

B. UNDAMPED ELECTRICAL OSCILLATOR

The differential equation of the electrical oscillator exposed to a triangular pulse force potential [9, 10] is given by

$$L\ddot{Q}(t) + \frac{1}{C}Q(t) = V_0 \left(1 - \frac{t}{t_1}\right)$$

Or

$$\ddot{Q}(t) + \omega_0^2 Q(t) = \frac{V_0}{L} \left(1 - \frac{t}{t_1}\right) \dots (5)$$

where $\omega_0 = \sqrt{\frac{1}{LC}}$, $V_0 \left(1 - \frac{t}{t_1}\right)$ is a triangular pulse potential and $Q(0) = 0$ and $\dot{Q}(0) = 0$.

The RT of equation (5) provides

$$q^2 \bar{Q}(q) - q^4 Q(0) - q^3 \dot{Q}(0) + \omega_0^2 \bar{Q}(q) = \frac{V_0}{L} q^3 \int_0^\infty e^{-qt} \left(1 - \frac{t}{t_1}\right) dt$$

Here $\bar{Q}(q)$ denotes the RT of $Q(t)$.

Put [11] $Q(0) = 0$ and $\dot{Q}(0) = 0$ and simplifying, we get

$$q^2 \bar{Q}(q) + \omega_0^2 \bar{Q}(q) = \frac{V_0}{L} \left\{ q^3 \int_0^{t_1} e^{-qt} \left(1 - \frac{t}{t_1}\right) dt + q^3 \int_{t_1}^\infty e^{-qt} (0) dt \right\}$$

$$q^2 \bar{Q}(q) + \omega_0^2 \bar{Q}(q) = \frac{V_0}{L} \left\{ q^3 \int_0^{t_1} e^{-qt} (1) dt - q^3 \int_0^{t_1} e^{-qt} \left(\frac{t}{t_1}\right) dt \right\}$$

$$\Rightarrow q^2 \bar{Q}(q) + \omega_0^2 \bar{Q}(q) = \frac{V_0}{L} \left\{ -q^2 [e^{-qt_1} - 1] + q^2 [e^{-qt_1}] + \frac{q}{t_1} [e^{-qt_1} - 1] \right\}$$

$$\Rightarrow q^2 \bar{Q}(q) + \omega_0^2 \bar{Q}(q) = \frac{V_0}{L} \left\{ q^2 + \frac{q}{t_1} e^{-qt_1} - \frac{q}{t_1} \right\}$$

$$\Rightarrow \bar{Q}(q) = \frac{V_0}{L} \left\{ \frac{q^2}{(q^2 + \omega_0^2)} + \frac{q}{t_1 (q^2 + \omega_0^2)} e^{-qt_1} - \frac{q}{t_1 (q^2 + \omega_0^2)} \right\}$$

$$\Rightarrow \bar{Q}(q) = \frac{V_0}{L} \left\{ \frac{q^4}{q^2 (q^2 + \omega_0^2)} - \frac{q^3}{t_1 q^2 (q^2 + \omega_0^2)} + \frac{q^3}{t_1 q^2 (q^2 + \omega_0^2)} e^{-qt_1} \right\}$$

$$\Rightarrow \bar{Q}(q) = \frac{V_0}{L} \left\{ \frac{q^2}{(\omega_0^2)} - \frac{q^4}{(\omega_0^2)(q^2 + \omega_0^2)} - \frac{q}{t_1 (\omega_0^2)} + \frac{q^3}{t_1 (\omega_0^2)(q^2 + \omega_0^2)} + \frac{q}{t_1 (\omega_0^2)} e^{-qt_1} - \frac{q^3}{t_1 (\omega_0^2)(q^2 + \omega_0^2)} e^{-qt_1} \right\}$$

Taking inverse RT, we have

$$Q(t) = \frac{V_o}{L} \left\{ \frac{1}{(\omega_0^2)} - \frac{\cos \omega_0 t}{(\omega_0^2)} - \frac{t}{t_1(\omega_0^2)} + \frac{\sin \omega_0 t}{t_1 \omega_0 (\omega_0^2)} \right\} + \frac{(t-t_1)}{t_1(\omega_0^2)} U(t-t_1) - \frac{\sin \omega_0(t-t_1)}{t_1 \omega_0 (\omega_0^2)} U(t-t_1)$$

$$\Rightarrow Q(t) = \frac{V_o}{L(\omega_0^2)} \left\{ 1 - \cos \omega_0 t - \frac{t}{t_1} + \frac{\sin \omega_0 t}{t_1 \omega_0} + \frac{(t-t_1)}{t_1} U(t-t_1) - \frac{\sin \omega_0(t-t_1)}{t_1 \omega_0} U(t-t_1) \right\}$$

$$\Rightarrow Q(t) = \frac{V_o}{1/C} \left(\frac{\sin \omega_0 t}{t_1 \omega_0} - \cos \omega_0 t + 1 - \frac{t}{t_1} + \frac{(t-t_1)}{t_1} U(t-t_1) - \frac{\sin \omega_0(t-t_1)}{t_1 \omega_0} U(t-t_1) \right)$$

$$\Rightarrow Q(t) = V_o C \left\{ \frac{\sin \omega_0 t}{t_1 \omega_0} - \cos \omega_0 t + \left(1 - \frac{t}{t_1} \right) - \left(1 - \frac{t}{t_1} \right) U(t-t_1) - \frac{\sin \omega_0(t-t_1)}{t_1 \omega_0} U(t-t_1) \right\} \dots (6)$$

Taking $V_o C = 500 \text{ coulomb}$, $\omega_0 = \frac{314 \text{ rad}}{\text{sec}}$, $t_1 = 1 \text{ sec}$, the graph of equation 6 is shown in figure below.

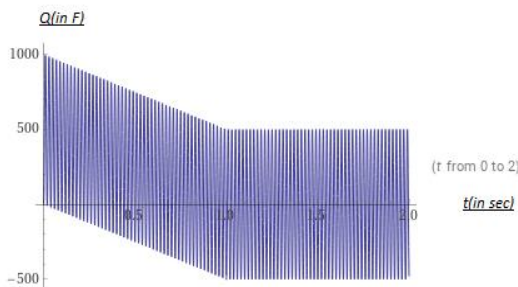


Figure 5: Response of an undamped electrical oscillator exposed to a triangular pulse potential.

For $t < t_1$,

$$Q(t) = V_o C \left\{ \frac{\sin \omega_0 t}{t_1 \omega_0} - \cos \omega_0 t + \left(1 - \frac{t}{t_1} \right) \right\} \dots (7a)$$

Taking $V_o = 230V$, $C = 1 \text{ micro farad}$, $t_1 = 0.1s$ and $\omega_0 = 314 \frac{\text{rad}}{s}$. The graph of equation (7a) is shown in figure 6.

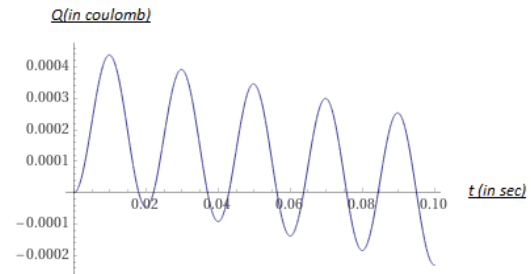


Figure 6: Response of an undamped electrical oscillator during the application of triangular pulse potential.

For $t > t_1$,

$$Q(t) = V_o C \left\{ \frac{\sin \omega_0 t}{t_1 \omega_0} - \cos \omega_0 t - \frac{\sin \omega_0(t-t_1)}{t_1 \omega_0} \right\} \dots (7b)$$

Taking $V_o = 230V$, $C = 1 \text{ micro farad}$, $t_1 = 0.1s$ and $\omega_0 = 314 \frac{\text{rad}}{s}$. The graph of equation (7b) is shown in Figure 7.

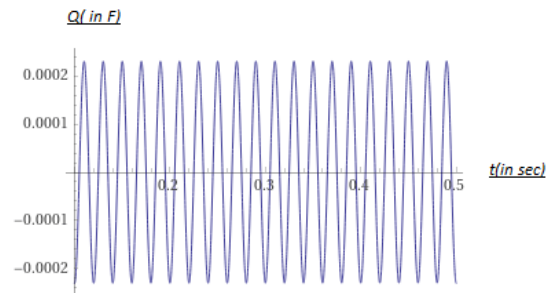


Figure 7: Response of an undamped electrical oscillator after the termination of triangular pulse potential.

III. DISCUSSION

It is clear from Figure 2, Figure 3 and Figure 4 that due to triangular pulse, the response (displacement) of an undamped mechanical oscillator exposed to a triangular pulse first increases towards right side (say) with large amplitude and then decreases and becomes zero and then increases towards left side with small amplitude and then decreases and becomes zero. Again, it repeats the same behavior again and again till

the effect of triangular pulse becomes zero, but with linearly decreasing amplitude towards right side and correspondingly linearly increasing amplitude towards the left side. As soon as the effect of triangular pulse becomes zero, the nature of displacement of an undamped mechanical oscillator exposed to a triangular Pulse suddenly becomes oscillatory with constant amplitude. It is clear from Figure 5, Figure 6 and Figure 7 that due to triangular pulse, the response (electric charge) of an undamped electrical oscillator exposed to a triangular pulse first increases in one direction with large amplitude and then decreases and becomes zero and then increases towards opposite side with small amplitude and then decreases and becomes zero. Again, it repeats the same behavior again and again till the effect of triangular pulse becomes zero, but with linearly decreasing amplitude in one direction and correspondingly linearly increasing amplitude in the other direction. As the triangular pulse ceases, the response of an undamped electrical oscillator exposed to a triangular Pulse suddenly becomes oscillatory with constant amplitude.

IV. CONCLUSION

In this paper, the response of SDOF systems such as an undamped mechanical oscillator as well as an undamped electrical oscillator exposed to a triangular pulse has been successfully determined by the integral Rohit transform (RT). It is clear from the discussion that even though the amplitude of triangular pulse is decreasing linearly, the response of SDOF systems is linearly decreasing in one direction and correspondingly increasing in other direction till the triangular pulse ceases. As the triangular pulse terminates, their response becomes oscillatory with constant amplitude. A new method is exploited for determining the response of SDOF systems (an undamped mechanical oscillator as well as an undamped electrical oscillator) exposed to a triangular pulse.

REFERENCES

- [1] Chopra, A.K. Dynamics of Structures: Theory and Applications to Earthquake Engineering. 4th Ed., Prentice Hall, 2011.
- [2] Paz, Mario, Structural Dynamics: Theory and Computation. 5th Ed., CBS Publishers, 2006.
- [3] Anil Kumar, Poonam, Ashok K Gupta, Effect of Damping on Shock Spectra of Impulse Loads, International Journal of Scientific & Engineering Research, 5, (2014)78-85.
- [4] Rohit Gupta, On novel integral transform: Rohit Transform and its application to boundary value problems, ASIO Journal of Chemistry, Physics, Mathematics and Applied Sciences (ASIO-JCPMAS), 4(2020)08-13.
- [5] Rohit Gupta, Inderdeep Singh, Analysis of one-way streamline flow between parallel plates via Rohit integral transform, International Journal of Trendy Research in Engineering and Technology, 6(2022) 29-32.
- [6] Rohit Gupta, Inderdeep Singh, and Ankush Sharma, "Response of an Undamped Forced Oscillator via Rohit Transform," International Journal of Emerging Trends in Engineering Research, 10,(2022)396-400.
- [7] Rohit Gupta, Mechanically Persistent Oscillator Supplied With Ramp Signal, Al-Salam Journal for Engineering and Technology, 2(2023) 112-115. DOI: 10.55145/ajest.2023.02.02.014
- [8] Rahul Gupta., Rohit Gupta, 'Response of Undamped Oscillators Exposed to Rectangular Pulse Force', *International Journal of Scientific Research in Physics and Applied Sciences*, 11(2023)10-14.
- [9] Rahul Gupta, Rohit Gupta, Dinesh Verma, Analysis of LCR Network Circuits with Exponential Sources by New Integral Transform Gupta Transform, Engineering and Scientific International Journal (ESIJ), 9(2022)7-29. DOI: 10.30726/esij/v9.i2.2022.92002
- [10] Deshpande A.S. 'Transient Analysis of R-L-C Series Circuit to Step voltage by Engineering Method', International Journal of Computational and Applied Mathematics, 9(2014)63-70.
- [11] A Sudhakar and Shyamamohan S Palli, Circuits and Networks Analysis and Synthesis, McGraw-Hill Education India Pvt, Ltd, 2010.