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ANALYSIS OF ONE-WAY STREAMLINE FLOW BETWEEN PARALLEL PLATES VIA ROHIT INTEGRAL TRANSFORM

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ABSTRACT

This paper illustrates the application of the Rohit integral transform for analyzing the one-way streamline flow between parallel plates directly without finding the general solution of a differential equation relating to the flow characteristic equation of the viscous liquid. Viscosity is the characteristic of a fluid (liquid) due to which viscous force becomes active when the fluid is in motion. This force opposes the relative motion of different layers of the fluid. This viscous force becomes active when the different layers of the fluid are operating with different velocities which leads to shearing stress between the layers of the operating fluid. In this paper, Rohit integral transform is applied for solving the differential equation relating flow characteristics of the viscous liquid to obtain the velocity distribution and shear stress distribution of a one-way streamline flow between the stationary parallel plates as well as between the parallel plates having a relative motion.

Index Terms
Streamline flow; Rohit integral transform; Parallel plates; Shear and Velocity distributions; viscous fluid.

I.INTRODUCTION

The steady flow of a viscous fluid (liquid) over a plane surface in the form of layers (in which the particles of the fluid operate in a regular and well-defined paths) of different velocities is known as streamline flow. Due to relative velocity, a velocity gradient exists between the two layers and as a result, the layers experience a shear stress. The flow of crude oil and highly viscous fluids through narrow passages, seepage through soils are some of the examples of the streamline flow. In such a flow the fluid (liquid) properties remain same in the directions perpendicular to the direction of flow of the fluid [1-5].

The Rohit transform of g(y) [6-11], y ≥ 0 is denoted by $G(q)$ and is given by

$G(q) = q^3 \int_0^{\infty} e^{-qy}\, g(y)dy$, provided the integral is convergent.

Here q is a real or complex parameter.

The RT of some elementary functions [8-9] are given by

- $R\{y^n\} = \frac{n!}{q^{n-2}}$, *where* $n = 0,1,2,3 \dots \dots$
- $R\{e^{by}\} = \frac{q^3}{q-b}$, $q > b$
- $R\{sinby\} = \frac{b\,q^3}{q^2+b^2}$, $q > 0$
- $R\{sinhby\} = \frac{b\,q^3}{q^2-b^2}$, $q > |b|$
- $R\{cosby\} = \frac{q^4}{q^2+b^2}$, $q > 0$
- $R\{coshby\} = \frac{q^4}{q^2-b^2}$, $q > |b|$
- $R\{\delta(y)\} = q^3$, where $\delta(y)$ is Dirac delta function.

The RT of derivatives [7-10] of g(y) with respect to y (y ≥ 0) are given by

$$R\left\{\frac{\partial g(y)}{\partial y}\right\} = qG(q) - q^3 g(0),$$

$$R\left\{\frac{\partial^2 f(y)}{\partial y^2}\right\} = q^2G(q) - q^4 g(0) - q^3 g'(0),$$

$$R\left\{\frac{\partial^3 f(y)}{\partial y^3}\right\} = q^3G(q) - q^5 g(0) - q^4 g'(0) - q^3 g''(0),$$

And so on.

II.MATERIAL AND METHOD

Flow Characteristic Equation

Consider a steady and uniform streamline flow of the viscous fluid (liquid) between the two flat parallel plates situated at a perpendicular distance L. Let the distance in which the fluid is flowing be represented by x and the distance which is normal to the flow of fluid and parallel to the plane of paper be represented

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by z such that the lower plate is situated at z = 0 and the upper plate is situated at z = L [4-5].

To derive the flow characteristic equation of operating viscous fluid, we consider a small fluid element of length dx, breadth dy and height dz. Due to viscous effects, there exists a relative velocity between any two adjacent layers of the viscous fluid. Due to this, a shear stress is set up between them. For steady flow, there will not be any shear stress on the vertical faces of the fluid element.

If the shear stress on the lower face OAGF of the fluid element is represented by τ and that on the upper face CBDE is $\left[\tau + \frac{\partial \tau}{\partial z}\, dz\right]$, then the shearing force on the fluid element is

$$\left[\tau + \frac{\partial \tau}{\partial z}\, dz\right] dx\, dy - \tau\, dx\, dy = \frac{\partial \tau}{\partial z}\, dx\, dy\, dz$$

If the pressure intensity on the face OCEF of the fluid (liquid) element is represented by P and that on the face ABDG is $\left[P + \frac{\partial P}{\partial x}\, dx\right]$, then pressure force on the fluid (liquid) element is

$$P\, dy\, dz - \left[P + \frac{\partial P}{\partial x}\, dx\right] dy\, dz = -\frac{\partial P}{\partial x}\, dx\, dy\, dz$$

For steady flow, the acceleration is zero and hence the sum of the shearing) force and the pressure force in the direction of flow of the fluid (liquid) i.e. the resultant force in the x-direction must vanish [5].

Thus

$$\frac{\partial \tau}{\partial z}\, dx\, dy\, dz - \frac{\partial P}{\partial x}\, dx\, dy\, dz = 0$$

Or

$$\frac{\partial \tau}{\partial z} = \frac{\partial P}{\partial x} \quad \ldots\ldots\ldots\ldots (1)$$

Since we are concentrating on pressure gradient only in the direction of liquid flow, we can replace the partial derivatives by total derivatives. Thus

$$\frac{d\tau}{dz} = \frac{dP}{dx} \quad \ldots\ldots\ldots\ldots (2)$$

This means that the shearing gradient in the direction normal to the flow of the viscous fluid is equal to the pressure gradient along the direction of liquid flow.

According to Newton's law of viscosity, the value of shear stress is given by

$$\tau = \mu\, \dot{U}(z) \quad \ldots\ldots.. (3)$$

Where $\dot{U}(z)$ represents the rate of change of velocity w.r.t. z and μ represents the coefficient of viscosity.

Put equation (3) in equation (2), we get

$$\mu\, \ddot{U}(z) = \frac{dP}{dx} \quad \ldots\ldots\ldots\ldots\ldots. (4)$$

Solution of the Flow Characteristic Equation

We will analyze the streamline flow of viscous fluid on the basis of following assumptions [3-5]:

i) There is no relative velocity of the fluid with respect to the surfaces of the plates.
ii) There are no end effects of the surfaces on the viscous fluid.
iii) $\frac{dP}{dx}$ is a constant in the x–direction.
iv) The flow is steady and incompressible and the properties of the fluid do not vary in the directions normal to the direction of flow of the fluid.

Now taking Rohit Transform of equation (4), we get

$$R[\mu\, \ddot{U}(z)] = \frac{dP}{dx}\, R[1]$$

This equation results

$$\mu[r^2\overline{U}(r) - r^4 U(0) - r^3\, \dot{U}(0)] = r^2 \frac{dP}{dx} \quad \ldots\ldots(5)$$

(a) For Streamline Flow Between Stationary (Fixed) Parallel Plates

Considering the flow of fluid between two parallel fixed plates, we can write the relevant boundary conditions as given below [5]:

At z = 0 and z = L, U = 0.

Applying boundary condition: $U(0)$ = 0, equation (5) becomes,

$$\mu[r^2\overline{U}(r) - r^3\, \dot{U}(0)] = r^2 \frac{dP}{dx} \quad \ldots\ldots\ldots.. (6)$$

In this equation, $\dot{U}(0)$ is some constant so let us substitute $\dot{U}(0) = \varepsilon$. Also, since $\frac{dP}{dx}$ is uniform, therefore, on putting $\frac{dP}{dx} = -\emptyset$, where $\emptyset$ a constant and negative sign is indicates that the pressure of fluid decreases in the direction of flow of the fluid.

Equation (3) becomes

$$\mu[r^2\overline{U}(r) - r^3\, \varepsilon\,] = -r^2\, \emptyset$$

Or

$$\overline{U}(r) = r\, \varepsilon - \frac{\emptyset}{\mu} \ldots\ldots\ldots\ \ldots\ldots\ldots\ldots.. (7)$$

Taking inverse Rohit transform of equation (7), we get

$$U(z) = \varepsilon\, z - \frac{\emptyset}{2\mu} z^2 \ldots\ldots\ldots.. (8)$$

Determination of the Constant ε

To find the value of constant ε, applying boundary condition: $U(L) = 0$, equation (8) provides,

$$0 = \varepsilon L - \frac{\emptyset}{2\mu}L^2$$

Upon rearranging and simplification of the above equation, we get

$$\varepsilon = \frac{\emptyset}{2\mu}L \text{ ……….. (9)}$$

Substitute the value of ε from equation (9) in equation (8), we get

$$U(z) = \frac{\emptyset}{2\mu}L\,z - \frac{\emptyset}{2\mu}z^2$$

Or

$$U(z) = \frac{\emptyset}{2\mu}[L\,z - z^2] \text{……… (10)}$$

Differentiating equation (10) w.r.t. z, we get

$$\dot{U}(z) = \frac{\emptyset}{2\mu}[L - 2z] \text{ …….. (11)}$$

For maximum velocity, $\dot{U}(z) = 0$

This results

$$z = \frac{L}{2} \text{ ……………. (12)}$$

Put the value of z from equation (12) in equation (10), we get

$$U_{max} = \frac{\emptyset}{2\mu}\frac{L^2}{4}$$

Or

$$U_{max} = \frac{\emptyset}{8\mu}L^2 \text{ ……. (13)}$$

The shear stress distribution is determined by the application of Newton's law of viscosity as

$$\tau(z) = \mu\,\dot{U}(z)$$

Using equation (8), we get

$$\tau(z) = \frac{\emptyset}{2}[L - 2z] \text{………. (14)}$$

At $z = \frac{L}{2}$ i.e. at the mid of the fixed parallel plates, $\tau\left(\frac{L}{2}\right) = \frac{\emptyset}{2}[L - 2\frac{L}{2}] = 0$ i.e. there is no shear stress even when there is constant pressure gradient.

At $z = 0$ i.e. at the surface of the lower fixed plate,

$$\tau(0) = \frac{\emptyset}{2}L$$

At $z = L$ i.e. at the surface of the upper fixed plate,

$$\tau(L) = -\frac{\emptyset}{2}L$$

For a particular case, when $\emptyset = 0, \tau(z) = 0$ i.e. there is no shear stress between the fixed parallel plates if there is no pressure gradient.

(b) For Streamline Flow Between Parallel Plates Having Relative Motion

Considering the flow of fluid (liquid) between the parallel flat plates such that the lower plate is fixed at z = 0 and upper plate is moving uniformly with velocity U_o relative to the lower fixed plate in the direction of flow of the fluid, we can write the relevant boundary conditions as given below [5]:

At z = 0, $U = 0$ and at z = L, $U = U_o$.

Applying boundary condition: $U(0) = 0$, equation (5) becomes,

$$\mu[r^2\overline{U}(r) - r^3\,\dot{U}(0)] = -r^2\,\emptyset \text{ ….. (15)}$$

In this equation, $\dot{U}(0)$ is some constant.

Let us substitute $\dot{U}(0) = \delta$,

Equation (15) becomes

$$\mu[r^2\overline{U}(r) - r^3\,\delta\,] = -r^2\,\emptyset$$

Or

$$\overline{U}(r) = r\,\delta - \frac{\emptyset}{\mu}\text{……………….. (16)}$$

Taking inverse Rohit transform [8-9] of equation (16), we get

$$U(y) = \delta\,z - \frac{\emptyset}{2\mu}z^2\text{………..….. (17)}$$

Determination of the Constant δ

To find the value of constant δ, applying boundary condition: $U(L) = U_o$, equation (17) provides,

$$U_o = \delta\,L - \frac{\emptyset}{2\mu}L^2$$

Upon rearranging and simplification of the above equation, we get

$$\delta = \frac{U_o}{L} + \frac{\emptyset}{2\mu}L \text{ …………... (18)}$$

Substitute the value of δ from equation (18) in equation (17), we get

$$U(z) = [\frac{U_o}{L} + \frac{\emptyset}{2\mu}L]\,z - \frac{\emptyset}{2\mu}z^2$$

Or

$$U(z) = \frac{U_o}{L}z + \frac{\emptyset}{2\mu}[L\,z - z^2] \text{ ……… (19)}$$

Differentiating equation (19) w.r.t. z, we get

$$\dot{U}(z) = \frac{U_o}{L} + \frac{\emptyset}{2\mu}[L - 2z]\text{………… (20)}$$

For maximum velocity, $\dot{U}(z) = 0$

This results

$$z = \frac{L}{2} - \frac{\mu U_o}{L\emptyset}\text{…………..…. (21)}$$

Put the value of z from equation (21) in equation (19) and simplifying, we get

$$U_{max} = \frac{\mu {U_o}^2}{L^2\,\emptyset} \ldots\ldots\ldots\ldots (22)$$

The shear stress distribution is determined by the application of Newton's law of viscosity as

$$\tau(z) = \mu\,\dot{U}(z)$$

Using equation (20), we get

$$\tau(z) = \frac{\mu U_o}{L} + \frac{\emptyset}{2}[L\ - 2z] \ldots\ldots (23)$$

At $z = \frac{L}{2}$ i.e. at the mid of the flow passage,

$$\tau\left(\frac{L}{2}\right) = \frac{\mu U_o}{L}$$

At $z = 0$ i.e. at the surface of the lower plate,

$$\tau(0) = \frac{\mu U_o}{L} + \frac{\emptyset}{2}L$$

At $z = L$ i.e. at the surface of the upper plate,

$$\tau(L) = \frac{\mu U_o}{L} - \frac{\emptyset}{2}L$$

For a particular case, when $\emptyset = 0$, $\tau(z) = \frac{\mu U_o}{L}$ i.e. the shear stress between the plates is not zero and having a constant value even if there is no pressure gradient.

III.CONCLUSION

In this paper, we have obtained the velocity distribution and shear stress distribution of a one-way streamline flow between stationary parallel plates as well as between parallel plates having a relative motion by solving the differential equation describing the flow characteristics of a viscous fluid via Rohit integral transform. Thus, Rohit integral transform has presented a powerful tool for obtaining the solution of the differential equation representing flow characteristic of viscous liquid without finding the general solution. It is concluded that, in the case of one-way streamline flow with constant pressure gradient between stationary parallel plates, the velocity distribution is maximum at the midway between the parallel plates and decreases parabolically with maximum value at the midway between the parallel plates to a minimum value at the lower fixed plate as well as at the upper fixed plate but the shear stress varies linearly with a minimum value at the midway between the parallel plates to a maximum value at the lower fixed plate as well as at the upper fixed plate. In the case of streamline flow with constant pressure gradient between parallel plates having a relative motion, the velocity distribution is parabolic with a minimum at the lower fixed plate but the shear stress varies linearly and at the midway between the parallel plates having a relative motion it is equal to the mean of the values of the shear stresses at the lower fixed plate and at the uniformly moving upper plate, and having a constant value even if there is no pressure gradient between parallel plates having a relative motion.

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